Design of Closed Loop Control for under damped Systems Using PICS and Magnitude Optimum for PID Controller

Anjum Sultana, Swapna Katam

Index Terms—PID control, Posicast, MOMI, DRMO, under damped systems, controller tuning.

INTRODUCTION

Most of the processes in practice are stable and can be controlled by various types of controller structures. The controller parameters are usually not critical and they can vary significantly to achieve stable response. However, some types of processes, like robot arms, disk-drive heads, cranes, power-system electronics and similar exhibit oscillatory behavior. Such systems require special attention, since stable response can be achieved in significantly smaller controller parameter space. Moreover, the mentioned systems usually require closed-loop response without or with a relatively small overshoot. Several tuning rules have been proposed so far for oscillatory systems. In general they require relatively precise process model obtained by process identification. One of the methods is called Posicast Input Command Shaping (PICS) proposed by Smith (1957). The main idea of the method is to split control signal into direct and delayed paths. When such control signal is applied to the process, the direct and delayed signal paths counteract and attenuate oscillations at the process output.

If signal splitter is placed inside the close-loop, it modifies the process transfer function by making a sum of undelayed and delayed process model. Most of the tuning methods cannot deal with modified transfer functions, especially if the process transfer functions are not precisely identified. Therefore, the PICS method is mostly used as a reference shaper outside the closed-loop configuration. Another tuning method for PID controllers, which can be used for moderately oscillatory systems, is Magnitude Optimum Multiple Integration (MOMI) method (Vrančić et al., 2001). However, the method fails to find appropriate controller parameters for highly oscillatory processes. On the other hand, the method requires either process time-

response (not necessarily step-response) or process model in order to calculate controller parameters according to the Magnitude Optimum (MO) criteria. This results in a relatively fast and non-oscillatory system closed-loop response. The MO method optimizes reference tracking. However, by using the MO method, the process poles could be cancelled by the controller zeros. This may lead to poor attenuation of load disturbances if the cancelled poles are excited by disturbances and if they are slow compared to dominant closed-loop poles. Poorer disturbance rejection performance can be observed when controlling low-order processes. This is one of the most serious drawbacks of the MO method. In process control, disturbance rejection is usually more important than superior tracking.

Recently, a modified method for tuning parameters of PI controllers, based on the magnitude optimum method, has been developed. Namely, the original magnitude optimum (MO) has been adjusted for improving disturbance rejection by using the so-called modified MO method or disturbance-rejection MO (DRMO) method. The results were encouraging, since disturbance rejection performance has been greatly improved, especially for lower-order processes. However, the modification of the magnitude optimum method is not limited only to the PI controllers. Recently, DRMO method has been applied to the PID controllers as well. However, since the structure of the PID controller is more complex, the calculation of the PID controller parameters could not be performed analytically as is the case for the PI controller. The PID controller parameters were calculated by using numerical methods.

Recently, the applicability of the MO method has been improved by using the concept of ‘moments’, which originated in identification theory (Ba Hli, 1954; Strejc, 1960; Rake, 1987). In particular, the process can be parameterized by subsequent (multiple) integrals of its input and output time-responses. Instead of using an explicit process model, the new tuning method employs the mentioned multiple integrals for the calculation of the PID controller parameters and is, therefore, called the “Magnitude Optimum Multiple Integration” (MOMI) tuning method (Vrančić, 1995; Vrančić et al., 1999). The proposed approach therefore uses information from a relatively simple experiment in a time-domain while retaining all the advantages of the MO method. The deficiency of the MO (and consequently of the MOMI) tuning method is that it is designed for optimizing tracking performance. This can lead to the poor attenuation of load disturbances (Astrom & Hagglund, 1995).

Disturbance rejection performance is particularly decreased for lower-order processes. This is one of the most serious disadvantages of the MO method, since in process control disturbance rejection performance is often more important than tracking performance. The mentioned deficiency has been recently solved by modifying the original MO criteria (Vrančić et al., 2004b; Vrančić et al., 2010). The modified criteria successfully optimized the disturbance rejection response instead of the tracking response. Hence, the concept of moments (multiple
integrations) has been applied to the modified MO criteria as well, and the new tuning method has been called the “Disturbance Rejection Magnitude Optimum” (DRMO) method (Vrančić et al., 2004b; Vrančić et al., 2010). The MOMI and DRMO tuning methods are not only limited to the self-regulating processes. They can also be applied to integrating processes (Vrančić, 2008) and to unstable processes (Vrančić & Huba, 2011). The methods can also be applied to different controller structures, such as Smith predictors (Vrečko et al., 2001) and multivariable controllers.

The main idea of this paper is to combine PICS and MOMI method into a new method for tuning oscillatory systems. Namely, PICS can be used in a usual way to obtain less oscillatory process response. Then, MOMI method can be applied to calculate the appropriate PID controller parameters. As will be shown in the paper, the controller parameters can be obtained either from the process time response or from the process transfer function. Moreover, either time-domain experiment or identification does not have to be repeated after calculating the PICS parameters.

II. PICS METHOD

Posicast is a feed forward control method that dampens oscillations in systems whose other transient specifications are otherwise acceptable. When properly tuned, the controlled system yields a transient response that has deadbeat nature. Consider a system having a lightly damped step response as shown in Fig. 1(a). The overshoot in the response can be described by two parameters. First, the time to the first peak is one half the undamped response period \(T_d\). Second, the peak value is described by \(1 + \delta\), where \(\delta\) the normalized overshoot, which ranges from zero to one is. Zero overshoot corresponds to critical damping. Posicast splits the original step input command into two parts, as illustrated in Fig. 1(b). The first part is a scaled step that causes the first peak of the oscillatory response to precisely meet the desired final value. The second part of the reshaped input is full scale and time-delayed to precisely cancel the remaining oscillatory response, thus causing the system output to stay at the desired value. Such is the idea behind “half-cycle Posicast,” which can be modeled using just the two parameters \(\delta\) and \(T_d\). The resulting system output is sketched in Fig. 1(c); the uncompensated output is also shown for comparison. Another description of half-cycle Posicast follows the example originally presented by Smith [1] and Cook [2]. Consider the problem of moving a load suspended by a cable attached to a gantry. The sequence of movements is illustrated in Fig. 2. In the uppermost frame A, the gantry and the load are both at position ‘1.’ The motion starts in the second frame B, with the gantry moving and then stopping at position ‘2,’ thus causing the load to swing toward position 3. In the third frame C, the load has swung past the gantry to position ‘3,’ and is about to swing back. Finally, in frame D, the gantry immediately moves to position ‘3,’ so that the load stays at position 3 without overshoot or oscillations.

A. The Analytical Model of Posicast

One block diagram interpretation of the half-cycle Posicast controller is shown in Fig. 3(a). The model has two forward paths. The upper path is that of the original, uncompensated command input. In the lower path, a portion of the original command is initially subtracted, so that the peak of the response will not overshoot the desired final value. Precisely a half cycle later, the command is fully restored to cancel oscillations and maintain the final value. The transfer function given by the function \(1 + P(s)\), where \(P(s)\) is given by

\[
P(s) = \frac{\delta}{1 + \delta} \left[ 1 - e^{-\frac{T_d}{2}} \right]
\]  

(1)

Posicast can be easily constructed in MATLAB’s SIMULINK environment by using the transport delay block. A sample diagram is shown in Fig. 3(b).
Fig 2 Explaining half-cycle Posicast using the gantry problem

(a) Transfer function form

(b) Simulink Diagram

Fig 3. Block diagrams for half cycle Posicast

Posicast Input Command Shaping method is defined for the following second-order process

\[ G_p(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]  

(2)

where \( \omega_n \) represents the natural frequency and \( \xi \) the damping factor of the process. The under-damped natural frequency is:

\[ \omega_d = \omega_n \sqrt{1 - \xi^2} \]  

(3)

Figure 4 represents typical time-response of the process (1) on unity-step input signal. Variable \( T_{pk} \) represents the peak time, which is half of oscillation period \( T_d \):

\[ T_{pk} = \frac{T_d}{2} = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} \]  

(4)

The overshoot \( d \) of the second-order process can be calculated from the following expression (Seborgetal., 1989):

\[ d = \frac{y_{pk} - y(\infty)}{y(\infty) - y(0)} = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \]  

(5)

The main concept of PICS method is to split the process input \( u \) signal into two parts, as shown in Figure 5. Transfer function of the PICS term is the following (Huey et al., 2008):

\[ G_{PICS}(s) = K_1 + (1 - K_1 e^{-\tau T_{dp}}) \]  

(6)

Gain \( K_1 \) and time delay \( T_{dp} \) are chosen so as to decrease oscillations of the process. For pure second-order process (1), the parameters are the following (Hung, 2007; Huey et al., 2008):

\[ K_1 = \frac{1}{1+\xi} \]  

(7)

\[ T_{dp} = T_{pk} \]  

The parameters \( K_1 \) and \( T_{dp} \) can also be estimated for the higher-order processes with one pair of complex poles. In this case, three successive peaks (minimums and maximums) from the process open-loop response should be measured, as shown in Figure 6. The Posicast parameters are then calculated from the amplitude and time difference between the peaks:

\[ K_1 \approx \frac{d_3}{d_1 + d_2} \]  

(8)

\[ T_{dp} = t_2 - t_1 \]
The efficiency of the PICS term will be illustrated on two process models.

**Case 1**
Consider the following second-order process model:

\[ G_p(z) = \frac{1}{1+0.5z^{-1}} \]  

(9)

According to expressions (2) - (5):

\[ T_{pk} = 3.24 \]

\[ \delta = 0.444 \]  

(10)

PICS term parameters are calculated from (8):

\[ K_1 = 0.693 \]

\[ T_{dp} = 3.24 \]  

(11)

Figure 7 shows the process open-loop step response without (broken line) and with PICS term (solid line). It can be seen that the PICS term is very efficient in reducing process oscillations and overshoots.

**Case 2**
Consider the following fourth-order process model:

\[ G_p(z) = \frac{1}{(1+0.5z^{-1})(1+z^{-2})} \]  

(12)

The open-loop response is shown in Figure 5 (see broken line). Since the process is of the higher order, measurement of the difference between the peaks should be performed (compare Figures 6 and 8):

\[ d_1 = 0.434 \]

\[ d_2 = 0.3148 \]

\[ T_{dp} = 3.16s \]  

(13)

The PICS term parameters are then calculated from (8):

\[ K_1 = 0.58 \]

\[ T_{dp} = 3.16 \]  

(14)

Figure 8 shows the process open-loop step response without (broken line) and with PICS term (solid line). Again, the efficiency of the PICS term can be clearly noticed. Since the PICS term significantly decreases the overshoot and oscillations, it might be beneficial to use it within the closed-loop configuration. In this case the controller has to be tuned for the following modified process:

\[ G_{pp}(z) = [K_1 + (1-K_1)e^{-T_{dp}z}]G_p(z) \]  

(15)

Unfortunately, most of the existing tuning methods for PID controllers are not defined for the above process type, since it consists of two additive terms. Moreover, one of the terms has additional pure time delay

**III. THE ORIGINAL MO METHOD**

The objective of the magnitude optimum (MO) method is to maintain the closed-loop magnitude response curve as flat and as close to unity for as large bandwidth as possible for a given plant and controller structure [2,8](See Figure 9). Such a controller results in a fast and non-oscillatory closed-loop time response for a large class of processes.
This technique can be found under other names such as modulus optimum [1] or Betragsoptimum [1, 3, 4], and results in a fast and non-oscillatory closed-loop time response for a large class of process models.

If $G_{cl}(s)$ is the closed-loop transfer function from the set point to the process output, the controller is determined in such a way that

$$
\frac{d^r |G_{cl}(j\omega)|}{d\omega^r} |_{\omega = 0} = 0 \\
$$

for as many $r$ as possible [1].

Let us assume that the actual process can be described by the following rational transfer function:

$$
G(s) = \frac{K_{PR} b_1 s^{n-1} + \cdots + b_m s^{n-m}}{1 + a_1 s + \cdots + a_n s^{n}} e^{-T_{del} s},
$$

where $K_{PR}$ denote the process steady-state gain, and $a_1$ to $a_n$ and $b_1$ to $b_m$ are the corresponding parameters ($m \leq n$) of the process transfer function, where $n$ can be an arbitrary positive integer value. $T_{del}$ denotes pure time delay.

In our case, the controller structure is chosen to be of the PID type (see Fig. 10) and is described by the following rational transfer function:

$$
G_c(s) = K + \frac{K_I}{s} + \frac{K_D s}{1 + T_f s},
$$

where $K$, $K_I$, and $K_D$ are proportional gain, integral gain, and derivative gain, respectively. Parameter $T_f$ is the first-order filter time constant which filters all controller terms instead of derivative term only (Vrancic et al., 2005). This controller structure permit us to treat the PID controller as an ideal “schoolbook” controller:

$$
G_{PF}(s) = \frac{1}{(1 + T_{pf})},
$$

while filter term can be considered as a part of the process:

$$
G_{PF}(s) = G_P(s) \frac{1}{(1 + T_{pf})}.
$$

This assumption significantly simplifies calculation of controller parameters. The PID controller parameters (21) are calculated from the following expression (Vrancic et al., 2001):

$$
\begin{bmatrix}
K_I \\
K_P \\
K_D
\end{bmatrix} = \begin{bmatrix}
-A_1 & A_0 & 0 \\
-A_2 & A_2 & -A_1 \\
-A_3 & A_4 & -A_2
\end{bmatrix}^{-1} \begin{bmatrix}
-0.5 \\
0 \\
0
\end{bmatrix}
$$

where $K_I$, $K_P$ and $K_D$ are integral, proportional and derivative controller gains, respectively. Parameters A0 to A5 are the so-called process characteristic areas (moments) which can be calculated in time-domain by integrating filtered process GPF (22) input and output signal during the process steady-state change:

$$
\begin{align*}
I_{u1} &= \int_{0}^{T} u(\tau) d\tau \\
I_{y1} &= \int_{0}^{T} y(\tau) d\tau \\
I_{u2} &= \int_{0}^{T} I_{u1}(\tau) d\tau \\
I_{y2} &= \int_{0}^{T} I_{y1}(\tau) d\tau
\end{align*}
$$

The areas can be calculated as follows:

$$
\begin{align*}
A_0 &= y_0(\infty); \\
y_1 &= A_0 I_{u1}(t) - I_{y1}(t) \\
A_1 &= y_1(\infty); \\
y_2 &= A_1 I_{u1}(t) - A_0 I_{u2}(t) + I_{y2} \\
A_2 &= y_2(\infty); \\
y_3 &= A_2 I_{u1}(t) - A_1 I_{u2}(t) + A_0 I_{u3}(t) - I_{y3}(t)
\end{align*}
$$

On the other hand, the areas can also be obtained directly from the process transfer function. If filtered process transfer function $G_{PF}$
(22) is described by the expression (17), the areas can be calculated as follows (Vrancic et al., 2001):

\[
\begin{align*}
A_0 &= K_{PP} \\
A_1 &= K_{PP} \left( a_1 - b_1 + T_{d1}\alpha y \right) \\
A_2 &= K_{PP} \left[ b_2 - a_2 - T_{d2}\alpha y \right] (26) \\
A_3 &= K_{PP} \left[ (-1)^{k+1}\alpha y \right] + \sum_{i=1}^{k} (-1)^{k+1} \frac{T_{d2}\alpha y b_{2i}}{2i} + \sum_{i=1}^{k} (-1)^{k+1} \frac{T_{d2}\alpha y c_{2i}}{2i}
\end{align*}
\]

Therefore, the controller parameters can be calculated either from non-parametric measurements of the process in time domain (not restricted to step-response) or from parametric process model (17). The MOMI tuning method usually results in a fast and non-oscillatory closed-loop responses for large set of process models. However, the MOMI method fails for some of oscillatory processes, where the calculated controller parameters give unstable closed-loop responses.

V. MOMI- PICS TUNING METHOD

The main idea of this paper is to apply PICS compensator before the process, as shown in Figure 5. Then the controller parameters can be calculated for the entire process with compensator \(G_{PP}(s)\). In time-domain, the areas can be calculated directly from the step-response of the process with PICS compensator (e.g. solid lines in Figures 7 and 8). If the process is already expressed by a transfer function, the characteristic areas of the process with PICS compensator can be calculated in the following way. The areas of two multiplied transfer functions:

\[
G_P(s) = G_{PP}(s) G_{PICS}(s)
\]

where characteristic areas of the filtered process \(G_{PP}(s)\) are denoted as \(A_i\) and the areas of the PICS term \(G_{PICS}(s)\) are denoted as \(A_{ipc}\), can be calculated as follows:

\[
\begin{align*}
A_{0PP} &= A_0 A_{0PC} \\
A_{1PP} &= A_0 A_{1PC} + A_1 A_{0PC} \\
A_{2PP} &= A_0 A_{2PC} + A_1 A_{1PC} + A_2 A_{0PC} \\
A_{3PP} &= A_0 A_{3PC} + A_1 A_{2PC} + A_2 A_{1PC} + A_3 A_{0PC}
\end{align*}
\]

The PICS compensator transfer function (6), when developed into infinite Taylor series, becomes:

\[
G_{PICS}(s) = 1 - (1 - K_f)sT_{dp} + (1 - K_f)\frac{s^2T_{dp}^2}{2!} - (1 - K_f)\frac{s^3T_{dp}^3}{3!} + \ldots
\]

By comparing expressions (29) and (17), the areas (26) of the PICS term are:

\[
\begin{align*}
A_{0PC} &= 1 \\
A_{1PC} &= (1 - K_f)T_{dp} \\
A_{2PC} &= (1 - K_f)\frac{T_{dp}^2}{2!} \\
A_{3PC} &= (1 - K_f)\frac{T_{dp}^3}{3!}
\end{align*}
\]

Inserting expression (30) into expression (28) gives us the characteristic areas of the process with PICS term:

\[
\begin{align*}
A_{1PP} &= A_1 + (1 - K_f)A_0 T_{dp} \\
A_{2PP} &= A_2 + (1 - K_f)\left[ A_1 T_{dp} + A_0 \frac{T_{dp}^2}{2!} \right] \\
A_{3PP} &= A_3 + (1 - K_f)\left[ A_2 T_{dp} + A_1 \frac{T_{dp}^2}{2!} + A_0 \frac{T_{dp}^3}{3!} \right]
\end{align*}
\]

The PID controller parameters can be calculated from (23) by replacing areas \(A_i\) with \(A_{ipc}\).

Let us now calculate the PID controller parameters for the same fourth-order process model as in case 2 (12). The chosen filter time constant of the PID controller was \(T_F=0.1s\). The characteristic areas of the process with PICS term are the following (31):

\[
\begin{align*}
A_{0PP} &= 1, A_{1PP} = 1.33, A_{2PP} = 2.097 \\
A_{3PP} &= 2.21, A_{4PP} = 1.75, A_{5PP} = 1.103
\end{align*}
\]

The PID controller parameter can be calculated according to expression (23) by replacing areas \(A_i\) by \(A_{ipc}\):

\[
\begin{align*}
K_I &= 0.41, K_P = 0.98, K_D = 0.64
\end{align*}
\]

The closed-loop time response on step-change of the set point is shown in Figure 11. The response on the reference change is smooth and without oscillations. If the closed-loop structure would not include the PICS term, the PID controller parameters can be calculated directly from the process areas (32). However, the calculated controller parameters would lead to unstable closed-loop response.
VI. EXAMPLES

Consider the following delayed third-order process model:

$$G_P(s) = \frac{e^{-\tau}}{(1+0.5s+2s^2)(1+0.5s)}$$

(35)

The open-loop response of the process is shown in Figure 12 (broken line). By measuring the peaks, the following parameters have been obtained:

$$\alpha_1 = 0.4024 \quad \alpha_2 = 0.2289 \quad t_{dp} = 4.51s$$

(36)

The PICS term parameters are calculated from (8):

$$K_1 = 0.637 \quad T_{dp} = 4.51$$

(37)

The open-loop response of the process with PICS term is shown in Figure 12 (solid line). It is obvious that the PICS term is efficient in reducing oscillations in time-response.

The open-loop response of the process with PICS term is shown in Figure 12 (solid line). It is obvious that the PICS term is efficient in reducing oscillations in time-response. The PID controller parameters have been calculated from the process parameters, a-priori chosen filter time constant $T_F=0.1$ and PICS term parameters according to procedure given in the previous section:

$$K_i = 0.28 \quad K_P = 0.69 \quad K_D = 0.48$$

(38)

The open-loop response of the process with PICS term is shown in Figure 12 (solid line). Similarly as in the previous cases, the PICS term efficiently reduce oscillations in time response.

The second example employs the sixth-order process model with minimum-phase zero:

$$G_{P2}(s) = \frac{(1-12s)e^{-\tau}}{(1+s)(1+2s^2)(1+0.5s)^4}$$

(39)

The open-loop response of the process is shown in Figure 14 (dashed line). By measuring the peaks, the following parameters have been obtained:

$$\alpha_1 = 0.3243 \quad \alpha_2 = 0.1845 \quad t_{dp} = 9.03s$$

(40)

The PICS term parameters are calculated from (8):

$$K_1 = 0.637 \quad T_{dp} = 9.03$$

(41)

The open-loop response of the process with PICS term is shown in Figure 14 (solid line). Similarly as in the previous cases, the PICS term efficiently reduce oscillations in time response.

The PID controller parameters have been calculated from the process parameters, a-priori chosen filter time constant $T_F=0.1$, and PICS term parameters according to procedure given in the previous section:

$$K_i = 0.104 \quad K_P = 0.479 \quad K_D = 0.659$$

(42)

The closed-loop time response on step-change of the set-point is shown in Figure 15 (solid line). The response on the reference change is again smooth with very small overshoot and without noticeable oscillations.
change is again smooth with very small overshoot and without noticeable oscillations. In order to test robustness of the proposed method, the estimated parameter \( T_{pk} \) has been reduced by 10%. Since the PICS term is not optimal, there are some residual oscillations in the open-loop response (see dash-dotted line in Figure 14). Due to modified PICS term, the controller parameters became:

\[
K_I = 0.098 \quad K_P = 0.348 \quad K_D = 0.361
\]  

(43)

The closed-loop time response is shown in Figure 15 (dashed line). It can be seen that response is still very good without significant increase of the overshoot.

**VII. CONCLUSIONS**

The purpose of this paper is to present tuning methods for PID controllers which are based on the Magnitude Optimum (MO) method. The MO method usually results in fast and stable closed loop responses. However, it is based on demanding criteria in the frequency domain, which requires the reliable estimation of a large number of the process parameters. In practice, such high demands cannot often be satisfied. The results of the experiments showed that the proposed approach with the Posicast term inside the closed-loop configuration can significantly stabilize the process. The PID controller parameters are calculated according to the modified process by using the modified MOMI tuning method.

Test on several process models showed that the proposed approach resulted in a graceful tracking performance even for higher-order processes with a couple of complex poles and non minimum zero. The method is relatively robust to change of posicast parameters. Calculation of Posicast parameters can be performed easily from the process open-loop response (not necessarily step response!) or from the process model. The controller parameters can be then calculated from posicast parameters and the process characteristic areas by appropriate modification of MOMI method, as given in the paper.

Our further research will be concentrated on optimisation of disturbance rejection performance and anti-windup solutions.

**VIII. REFERENCES**


**AUTHORS BIOGRAPHY**
